MIDTERM

- This is the midterm for Math S1201: Calculus 3, Summer 2018.
- The use of class notes, book, calculator is not allowed. However, you may use a double sided A4 sized formula sheet.
- You have 95 minutes.
- Write your solutions in the space provided. Continue on the back if you need more space.
- Do not forget to write your name and UNI on the exam sheet and the notebook provided.

1. True-False. *Circle* T if you think the statement is true, otherwise *circle* F, you DON'T need to justify your answer. (10 points in total, 2 points each)

(1) $|\vec{u} - \vec{v}| = |\vec{v}| - |\vec{u}|$.

T F

(2) There is a vector $\vec{v}$ such that $\vec{v} \times \langle 1, 1, 1 \rangle = \langle 1, 2, -3 \rangle$.

T F

(3) The surface described by the equation $x^2 - y^2 - z^2 = -1$ is a hyperboloid of two sheets.

T F

(4) Let $\vec{a}(t)$, $\vec{b}(t)$ and $\vec{c}(t)$ be three vector valued functions. Then

$$\frac{d}{dt}(\vec{a} \times (\vec{b} \times \vec{c})) = \vec{a}' \times (\vec{b}' \times \vec{c}').$$

T F

(5) The curvature of a circle of radius 2 is a constant which is equal to 2.

T F

2. In the following, let $\vec{v} = \langle 1, 1, 1 \rangle$ and $\vec{w} = \langle -3, 1, -5 \rangle$.

(a) (5 points) Compute $\vec{v} \times \vec{w}$.
(b) (5 points) Compute the vector projection $\text{proj}_{\vec{w}} \vec{v}$.
(c) (5 points) Compute cosine of the *acute* angle between $\vec{v}$ and $\vec{w}$. (2 more subquestions on the back)

(d) (5 points) Find an equation describing a plane which is parallel to $\vec{v}$ and $\vec{w}$ and passing through the point $(1, 2, 3)$.

(e) (5 points) Find the distance between the plane found in (d) and the origin.

3. Let C be the curve on the xy-plane defined by $x^2 - 2y^2 = 1$. Consider rotating C around the x-axis, resulting a rotation surface in the 3-dimensional space. Denote this surface by S.

(a) (10 points) Find an polar equation of C (as a curve on the xy-plane) of the form $r = f(\theta)$, here we adopt the convention that x-axis is the polar axis.

(b) (10 points) Find an equation of S. (Hint: a point P is on the surface S if and only if there is a point Q on the curve C such that (1) P and Q have the same x-coordinate and (2) P and Q have the same distance to x-axis.)

(c) (5 points) Which class does S fall into in our classification of quadric surfaces? You don't need to justify your answer.

(d) (10 points) How many lines are there lying on *another surface*[1] with equation $x^2 - y^2 - z^2 = 1$ and passing through $(1, 0, 0)$? Write down their equations if you think there is any, justify your answer if you think there isn't any.

[1]Note that this surface is unrelated to the surface S

4. Consider the curve with position vector function $\vec{r}(t) = \sin(t)\,\vec{i} + t\,\vec{j} + \cos(t)\,\vec{k}$.
(a) (5 points) Find the unit tangent vector function $\vec{T}(t)$.

(b) (5 points) Find the curvature $\kappa(t)$ of this curve.

(c) (5 points) Find the unit normal vector function $\vec{N}(t)$ and the unit binormal vector function $\vec{B}(t)$.

(d) (5 points) Reparametrize this curve by arc length s such that $\vec{r}(s)|_{s=0} = \langle 0, \pi, -1 \rangle$.

5. A particle is moving in the space with position vector function $\overrightarrow{r}(t)$ with initial speed $v(0) = 1$. Suppose we know its acceleration is given by $\overrightarrow{a}(t) = e^t\overrightarrow{T}(t) + e^t\overrightarrow{N}(t)$. Compute the normal component of the third derivative of $\overrightarrow{r}(t)$ (i.e., compute $\overrightarrow{r}(t)''' \cdot \overrightarrow{N}(t)$). (10 points)

Bonus question: write down a curve parametrized by arc length with curvature function given by $\kappa(s) = s$. (10 points, no partial credit)